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LETTER TO THE EDITOR

ε-expansion for the critical exponents of a vector spin glass

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Abstract. The ε-expansion for the critical exponents of a vector spin glass is calculated to $O(\varepsilon^3)$ ($\varepsilon = 6-d$).

Recent Monte Carlo calculations (Ogielski and Morgenstern 1984, Young and Bhatt 1984) and large cell renormalisation group calculations (McMillan 1984, Bray and Moore 1984) have suggested that the Ising spin glass may have a phase transition at finite temperature in three dimensions. This has led us to look again at the field theory model of a vector spin glass due to Harris *et al* (1976). In particular we have extended the ε-expansions for critical exponents from order ε^2 (Elderfield and McKane 1978) to order ε^3. We hope that in the future the high-order behaviour of these expansions can be evaluated so that they may be resummed using the techniques of Le Guillou and Zinn-Justin (1977, 1980) and Vladimirov *et al* (1979) to give numerical results in three dimensions.

The Hamiltonian for the m-vector spin glass model of Harris *et al* (1976) can be written

$$H = \int d^d x \left(\frac{1}{4} \sum_{\substack{\alpha \neq \beta \\ i,j}} [(\nabla Q_{ij}^{\alpha\beta}(x))^2 + m_0^2 (Q_{ij}^{\alpha\beta}(x))^2] + \frac{g}{3!} \sum_{\substack{\alpha \neq \beta \neq \gamma \\ i,j,k}} Q_{ij}^{\alpha\beta}(x) Q_{jk}^{\beta\gamma}(x) Q_{ki}^{\gamma\alpha}(x) \right) \quad (1)$$

where

$\alpha, \beta, \gamma, \ldots, n$ label the n replicas

$i, j, k = 1, \ldots, m$ label the spin components

and $Q_{ij}^{\alpha\beta}(x) = Q_{ji}^{\beta\alpha}(x)$.

We shall use Greek superscripts to label replicas and Roman subscripts to label spin components throughout this letter.

Before we describe in detail our ε-expansion calculation we consider whether the cubic theory with Hamiltonian (1) is well defined. In general a cubic field theory is not well defined (McKane 1979) but in two special cases, the Yang–Lee edge singularity (Kirkham and Wallace 1979) and the percolation problem (Houghton *et al* 1978), it has been shown that the theory is well behaved with oscillatory behaviour at high order in the ε-expansion. This is due to the taking of some 'unphysical' limit: in the Yang–Lee edge this is the imaginary coupling constant and in the percolation problem it is an $n \to 0$ limit. For the spin glass model it has not been shown that the ε-expansion has oscillatory behaviour at high order, however we believe that the 'unphysical' limit of zero replicas will make the theory stable. We should also consider the relevance of the higher-order terms in Q which arise when the field theory is derived from the lattice model. Elderfield and McKane (1978) investigated this problem by calculating the anomalous dimensions of the quartic terms to one-loop order. They found that the quartic interaction remained irrelevant down to three dimensions for the $m=3$ Heisenberg case but that this was not true for the Ising ($m=1$) or XY ($m=2$) models.

We now proceed to the description of our ε-expansion calculation. In de Alcantara Bonfim *et al* (1980, 1981 to be referred to as I) a general cubic field theory was considered with Hamiltonian

$$H=\int \mathrm{d}^d x\left(\frac{1}{2}\sum_i [(\nabla\phi_i(x))^2+m_0^2\phi_i^2(x)]+\frac{g}{3!}\sum_{i,j,k} d_{ijk}\phi_i(x)\phi_j(x)\phi_k(x)\right). \tag{2}$$

If we can write our spin glass Hamiltonian (1) in this form, we can read off the ε-expansions directly from I. Following Elderfield and McKane (1978) we define a tensor

$$F_{abcd}^{\alpha\beta\gamma\delta}=\tfrac{1}{2}[\delta^{\alpha\gamma}\delta^{\beta\delta}\delta_{ac}\delta_{bd}+\delta^{\alpha\delta}\delta^{\beta\gamma}\delta_{ad}\delta_{bc}-T^{\alpha\beta\gamma\delta}(\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc})] \tag{3}$$

where T is defined by

$$T^{\alpha\beta\gamma\delta}=\begin{cases}1 \text{ if } \alpha=\beta=\gamma=\delta\\ 0 \text{ otherwise.}\end{cases}$$

The tensor F has the following useful properties

$$F_{abcd}^{\alpha\beta\gamma\delta}=F_{cdab}^{\gamma\delta\alpha\beta}=F_{abdc}^{\alpha\beta\delta\gamma}$$

$$F_{abcd}^{\alpha\beta\gamma\delta}F_{cdeh}^{\gamma\delta\varepsilon\eta}=F_{abeh}^{\alpha\beta\varepsilon\eta}$$

where we have now introduced the Einstein summation convention. We can then write the Hamiltonian (1) in terms of free sums as

$$H=\int \mathrm{d}^d x\Big[\tfrac{1}{4}(\nabla Q_{ab}^{\alpha\beta}(x)\nabla Q_{cd}^{\gamma\delta}(x)+m_0^2 Q_{ab}^{\alpha\beta}(x)Q_{cd}^{\gamma\delta}(x))F_{abcd}^{\alpha\beta\gamma\delta}$$
$$+\frac{g}{3!}F_{abr_1r_2}^{\alpha\beta\rho_1\rho_2}F_{bcr_3r_4}^{\beta\gamma\rho_3\rho_4}F_{car_5r_6}^{\gamma\alpha\rho_5\rho_6}Q_{r_1r_2}^{\rho_1\rho_2}(x)Q_{r_3r_4}^{\rho_3\rho_4}(x)Q_{r_5r_6}^{\rho_5\rho_6}(x)\Big]. \tag{5}$$

The free propagator for the theory is then

$$\frac{2}{q^2+m_0^2}F_{abcd}^{\alpha\beta\gamma\delta} \tag{6}$$

and the two-point vertex function

$$\Gamma_{abcd}^{\alpha\beta\gamma\delta}(q,m_0)=\Gamma^2(q,m_0)F_{abcd}^{\alpha\beta\gamma\delta}$$
$$=(q^2+m_0^2)F_{abcd}^{\alpha\beta\gamma\delta}-2F_{abcd}^{\alpha\beta\gamma\delta}\,\Sigma\,(q,m_0) \tag{7}$$

where $\Sigma\,(q,m_0)$ is the sum of self energy diagrams. If we consider the one loop contribution to Σ shown in figure 1 and specialise to the Ising case then the contribution of this diagram is

$$2F^{\rho_1\rho_2\rho_3\rho_4}2F^{\rho_5\rho_6\rho_7\rho_8}F^{\alpha\beta\delta_1\delta_2}F^{\rho_1\rho_2\delta_2\delta_3}F^{\rho_7\rho_8\delta_3\delta_1}F^{\gamma\delta\delta_4\delta_5}F^{\rho_3\rho_4\delta_5\delta_6}F^{\rho_5\rho_6\delta_6\delta_4}$$
$$\times\tfrac{1}{2}g^2\int\frac{\mathrm{d}^d p}{(p^2+m_0^2)[(q-p)^2+m_0^2]}. \tag{8}$$

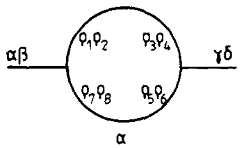

Figure 1. The one-loop Feynman graph contributing to the self energy for the spin glass. The labelling denotes the replica labels on the propagators.

After summing over all the repeated indices this becomes

$$(n-2)F^{\alpha\beta\gamma\delta}\tfrac{1}{2}g^2\int\frac{\mathrm{d}^dp}{(p^2+m_0^2)[(q-p)^2+m_0^2]}.$$

So for the Ising spin glass

$$\Gamma^2(q, m_0)=(q^2+m_0^2)-2(n-2)\tfrac{1}{2}g^2\int\frac{\mathrm{d}^dp}{(p^2+m_0^2)[(q-p)^2+m_0^2]}+\mathrm{O}(g^4). \tag{9}$$

The equivalent expression from I for the model (2) is

$$\Gamma^2(q, m_0)\delta_{ij}=(q^2+m_0^2)\delta_{ij}-\tfrac{1}{2}\alpha g^2\delta_{ij}\int\frac{\mathrm{d}^dp}{(p^2+m_0^2)[(q-p)^2+m_0^2]}+\mathrm{O}(g^4) \tag{10}$$

and hence we identify

$$\alpha=2(n-2)\qquad\text{for the Ising spin glass.}$$

The identification of the other tensor contractions β, γ, δ, λ, defined in I is easier as these are all associated with diagrams which contribute to the 3-point function (see figure 2). In the spin glass we define $\Gamma^3(q_i, m_0)$ by

$$\Gamma^{\alpha\beta\gamma\delta\mu\nu}_{abcdmn}(q_i, m_0)=\Gamma^3(q_i, m_0)F^{\alpha\beta\rho_1\rho_2}_{abr_1r_2}F^{\gamma\delta\rho_2\rho_3}_{cdr_2r_3}F^{\mu\nu\rho_3\rho_1}_{mnr_3r_1} \tag{11}$$

which is to be compared with

$$\Gamma^{ijk}(q_i, m_0)=d_{ijk}\Gamma^3(q_i, m_0)$$

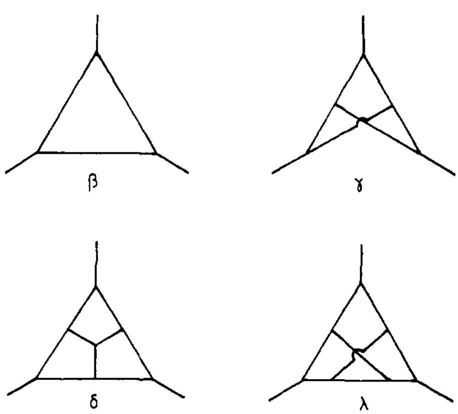

Figure 2. The Feynman graphs which correspond to the independent tensor contractions which arise in the 3-point function.

in order to calculate the β, γ, δ, λ. Beyond the two-loop level it becomes impracticable to contract the F's by hand and the Schoonschip† program was used. This made it possible to calculate the tensor contractions for the m-vector system. The results are

$$\alpha = 2(nm - 2m), \qquad \beta = 1 + nm - 3m, \qquad \gamma = 2 + 6nm - 18m,$$
$$\delta = -1 + n(1 + 30m - 10m^2 + 17m^3) + n^2(3m^2 - 6m^3) + n^3m^3 - 77m + 7m^2 - 23m^3,$$
$$\lambda = n(26m - 28m^2) + 5n^2m^2 - 62m + 50m^2. \tag{13}$$

Substituting these values into the results given in I and taking the $n \to 0$ limit:

$$\begin{aligned}
\eta = {} & -\frac{1}{3}\frac{m\varepsilon}{(2m-1)} + \frac{\varepsilon^2}{4^3(2m-1)^3}\left(-\frac{104}{9}m + \frac{2752}{27}m^2 - \frac{88}{9}m^3\right) \\
& + \frac{\varepsilon^3}{4^5(2m-1)^5}\left[\zeta(3)\left(-\frac{1024}{3}m - \frac{33280}{3}m^2 + 27136m^3 - 7168m^4\right)\right. \\
& \left. -\frac{4096}{27}m + \frac{320}{3}m^2 - \frac{2056544}{243}m^3 + \frac{144832}{81}m^4 - \frac{25376}{27}m^5\right] \\
\nu^{-1} - 2 + \eta = {} & \frac{-2m}{(2m-1)}\,\varepsilon + \frac{\varepsilon^2}{4^3(2m-1)^3}\left(-48m + \frac{1312}{3}m^2 + \frac{1840}{9}m^3\right) \\
& + \frac{\varepsilon^3}{4^5(2m-1)^5}\left(\zeta(3)(-3072m - 55808m^2 + 130560m^3\right. \\
& - 10240m^4 - 6144m^5) - 704m - \frac{42496}{9}m^2 \\
& \left. - 23936m^3 - \frac{1554176}{81}m^4 - \frac{366784}{27}m^5\right).
\end{aligned} \tag{14}$$

$$\zeta(3) = \sum_{n=1}^{\infty} \frac{1}{n^3} = 1.202\ldots$$

In the Ising $m = 1$ case these become

$$\begin{aligned}
\eta &= -0.3333\varepsilon + 1.2593\varepsilon^2 + 2.5367\varepsilon^3 \\
\nu^{-1} - 2 + \eta &= -2\varepsilon + 9.2778\varepsilon^2 + 4.2336\varepsilon^3.
\end{aligned} \tag{15}$$

In the Heisenberg $m = 3$ case the exponents are

$$\begin{aligned}
\eta &= -0.2\varepsilon + 7.7333 \times 10^{-2}\varepsilon^2 - 7.8127 \times 10^{-2}\varepsilon^3 \\
\nu^{-1} - 2 + \eta &= -1.2\varepsilon + 1.164\varepsilon^2 - 1.4735\varepsilon^3.
\end{aligned} \tag{16}$$

We notice that to this order in ε the terms show oscillatory behaviour in the Heisenberg case but not in the Ising case. A proper calculation of the high-order behaviour will show whether this is significant.

† SCHOONSCHIP is an algebraic manipulation program written by M Veltman. For further information see Strubbe (1974).

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References

Bray A J and Moore M A 1984a *J. Phys. C: Solid State Phys.* **17** L463
—— 1984 *Phys. Rev.* B to appear
de Alcantara Bonfim O F, Kirkham J E and McKane A J 1980 *J. Phys. A: Math. Gen.* **13** L247
—— 1981 *J. Phys. A: Math. Gen.* **14** 2391
Elderfield D J and McKane A J 1978 *Phys. Rev.* B **18** 3730
Harris A B, Lubensky T C and Chen J H 1976 *Phys. Rev. Lett.* **36** 415
Houghton A, Reeve J S and Wallace D J 1978 *Phys. Rev.* B **17** 2956
Kirkham J E and Wallace D J 1979 *J. Phys. A: Math. Gen.* **12** L47
Le Guillou J C and Zinn-Justin J 1977 *Phys. Rev. Lett.* **39** 95
—— 1980 *Phys. Rev.* B **21** 3976
McKane A J 1979 *Nucl. Phys.* B **152** 166
McMillan W L 1984a *Phys. Rev.* B **30** 476
—— 1984b Preprint
Ogielski A T and Morgenstern I 1984 *Preprint*
Strubbe H 1974 *Comp. Phys. Commun.* **8** 1
Vladimirov A A, Kazakov D I and Tarasov D V 1979 *Sov. Phys.-JETP* **50** 521
Young A P and Bhatt R H 1984 *Preprint*